The Existence and Uniqueness of the Solution of a Diffusive Predator-Prey Model with Omnivory and General Nonlinear Functional Response

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Introduction

The dynamic interaction between predators and their prey has become a prominent issue in both ecology and mathematical ecology in recent times, owing to its universal occurrence and significance in population dynamics. Over the course of nearly thirty years, studies on predator-prey models have been established, and in light of observations and laboratory trials, more realistic models have been constructed (see [1-9]). Three species' predator-prey systems have been the subject of intriguing and striking research since the 1970s[10–14]. For instance, using the same biotic resource, Safuan [14] examined the Lesie-Gower predator-prey model dx x = r1x(1 —~) — axy, ◻◻ dt pz.

dt qz dz

= z(c — dx — ey), dt, where r1, r2, a, b, c, d, e, p, and q are positive constants and populations of prey, predator, and biotic resource, respectively, are represented by functions x(t), y(t), and z(t). For additional information about the biological background of system (1.1), see [14] and the sources listed therein.

However, it is evident from the above model that the carrying capacities of the predator and prey are correlated with the population size of the biotic resource, meaning that the carrying capacities of the two species are dependent on the amount of biotic resource. At low densities, it exhibits slightly unique behaviour, which makes it impossible to linearize the model at the border equilibria. Thus, it is impossible to study the linear stability of boundary equilibria. In fact, this singularity adds a great deal to the model's diversity of dynamics and makes system analysis quite challenging. Actually, prey x and predator y will go to another population, but its expansion will be restricted, if this preferred food z is extremely insufficient. The carrying capacity of the model (1.1) is increased by a positive constant to produce the following modified Lesie-Gower omnivorous predator-prey model

(1.4)

r_2 e

Individuals in a species undergo continual spatial distribution changes as a result of a variety of factors during their evolutionary process. Consequently, it has been acknowledged that the spatial aspect of ecological interactions plays a significant role. Several spatial effects have been included to population models in recent years. Many studies extend the predator-prey model of ODE to the corresponding diffusive predator-prey model by integrating the diffusion components, taking into account the natural diffusion and inhibitory impact (see[15-36]).

Through the incorporation of diffusion into model (1.1) in [31], Jau examined the subsequent nonlinear diffusive Lesie-Gower model of predator-prey

where the densities of the prey, predator, and biotic resource at time t and location x ∈ Ω are represented by the variables u(t, x), v(t, x), and w(t, x), respectively. ∂Ω is C1−class, Ω is a bounded open set in Rn, and the Holder continuous functions on Ω are u0(x), v0(x), and w0(x). The boundary ∂Ω's outward unit normal vector is represented by ν. According to the homogeneous Neumann border conditions, there is no population movement over the boundary, indicating that the predator-prey system is self-contained. He looked into whether there was a unique solution for system (1.3), and as a consequence, he came to the following conclusion[31].

Theorem A Suppose that constants *ε, α, β, M, N* and *K* satisfy

$||0 < \varepsilon \leq \min w_0(x), \alpha \geq ||w_0||_\infty, \beta \geq c, M \geq \max\{◻u_0||_\infty, p\alpha e^{\beta T}\},$ $_{x\in\bar{\Omega}}$

$||N \geq \max\{◻v_0||_\infty, \underline{r_2 + bM}\, q\alpha e^{\beta T}, 1$

$(c - dM)\}, K \geq dM + eN - c,$

Then, (0, 0, εe−Kt) ≤ (u, v, w) ≤ (M, N, αeβt) and the system (1.3) has a unique solution (u, v, w) on [0, T] × Ω¯.

The three species food web models of the predator-prey type that we will be focusing on in this research have an omnivorous apex predator, which is characterised as feeding on many trophic levels. This is actually a general component of terrestrial or marine food web ecological systems. As an illustration, consider the following: species w are plants, species u are herbivores, and species v eat both plants and other herbivores. As an additional illustration, tiny animals like lizards and birds devour a lot of spiders and herbivorous insects. Additional examples can be found in the intricate marine food web systems. The terms "trophic level omnivory," "intraguild predation," "higher order predation," and "hyperpredation" have all been used to describe this phenomenon[37]. Inspired by the previous research, we would want to examine the following diffusive modified Lesie-Gower predator-prey model with omnivory and general nonlinear functional response ∂u˜˜u˜ ∂v by adding the diffusion into the model (1.2).

u(t, x), v(t, x), and w(t, x) are the densities of the prey, predator, and biotic resource at time t and position x ∈ Ω, respectively. u(0, x) = u0(x), v(0, x) = v0(x), and w(0, x) = w0(x), x ∈ Ω. With a general nonlinear functional response ϕ(u), the predator devours the prey and contributes to its growth at a rate bϕ(u). The following presumptions are applied to the function ϕ(u), which Georgescu and Morosanu thoroughly examined in [38].

(G) For u ∈ R+, where L ≥ 0, ϕ(u) of the C1−class is rising on R+, ϕ(0) = 0, and 0 ≤ ϕ′(u) < L.

It should be noted that if function ϕ(u) represents the Holling type II functional response, that is, ϕ(u) = au/(1 + hu), where an is the resource and intermediate consumer search rates and h is the corresponding clearance rate, or the search rate multiplied by the (ostensibly constant) handling time, then hypothesis (G) is satisfied.

In order to demonstrate the existence and uniqueness of the solution for the system (1.5) using the methods of the upper and lower solutions [39] and the semigroup theory [40, 41], we will expand on the analysis methodology of Jau [31] in this study. The remainder of the paper is organised as follows. We shall present the idea of the higher and lower solutions in the remaining sections of this part. Assuming the existence of the lower solution and the upper solution, u˵ = (u˵, v˜, w˵), in Section 2.

For the problem (1.5), uˆ = (uˆ, vˆ, wˆ), we will demonstrate the existence and uniqueness of the solution on the sector ▨uˆ, u˵▩ ≡ {u= (u, v, w) ∈ C(D¯T):uˆ ≤ u ≤ u˵}. Two upper and lower solutions to problem (1.5) on [0, T]×Ω¯, where T is an arbitrary positive number, are provided in Section 3. Next, we establish that the solution (u, v, w) to problem (1.5) on [0, T] × Ω¯ exists and is unique. Conclusions are provided at the end of the paper in Section 4.

Let u1 = u, u2 = v, u3 = w, u1,0 = u0, u2,0 = v0, u3,0 = w0, L1 = d1∆, L2 = d2∆, L3 = 0, B = ∂, and so on in order to simplify the notations of the system (1.5).

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u

2

$f_1(u_1, u_2, u_3) = r_1u_1 -$ $\quad - \varphi(u$

r

2

u

2

$f_2(u_1, u_2, u_3) = r_2u_2 -$ $\quad + b\varphi(u$

$f_3(u_1, u_2, u_3) = cu_3 - du_1u_3 - eu_2u_3.$

(1.6)

Assuming that T is an arbitrary positive number and that DT = (0, T] ×Ω and ST = (0, T] × ∂Ω, system (1.5) can be expressed as (ui)t − Liui = fi(u1, u2, u3), where i = 1, 2, 3, and DT = (0, T] ×Ω.

In ST, i = 1, 2 ui(0, x) = ui,0(x), in Ω, i = 1, 2, 3. Bui(t, x) = 0.

regarding every (u1, u2, u3) ∈ J1 × J2 × J3. According to this, for every (u1, u2, u3) ∈ J1 × J2 × J3, f1 is monotone nonincreasing in u2 and monotonous nondecreasing in u3, f2 is monotone nondecreasing in u1 and monotone nondecreasing in u3, and f3 is monotone nonincreasing in u1 and monotone nonincreasing in u2. For each i = 1, 2, 3, where ai + bi = 2, let u = (u1, u2, u3) and fi(u)= fi(ui,[u]ai,[u]bi). Then, fi is monotone nonincreasing in [u]bi and monotone nondecreasing in [u]ai. The coupled upper and lower solutions of the system (1.7) are then defined as follows (see[39]).

Definition 1.1. If u˜1, u˜2, uˆ1, uˆ2 ∈ C(D¯T) ∩ C1,2(DT), u˜3, uˆ3 ∈ C(D¯T) ∩ C1,0(DT), and u˜ ≥ uˆ (i.e., u˜2 ≥ uˆ1) with uˆ3(t, x) > 0 in DT = [0, T] × Ω¯, and (u˜i)t − Liu˜i − fi(u˜i, [u˜]ai, [uˆ]bi) ≥ 0, in DT, i = 1, 2, 3.

(uˆi)t − Liuˆi − fiIn DT, where i = 1, 2, 3, (uˆi, [u˜]ai, [u˜]bi) ≤ 0.

On ST, i = 1, 2 u˜i(0, x) ≥ ui,0(x) ≥ uˆi(0, x), in Ω, i = 1, 2, 3. Bu˜i(t, x) ≥ 0 ≥ Buˆi(t, x).(1.9)

We define the sector ▨uˆ, u˜▆ ≡ {u= for a given pair of connected upper and lower solutions, u˵, uˆ.

(v, w) ∈ C(D¯T): uˆ {≤ u ≤ u˜}. Suppose that c1 = 2r1u˜1 + Lu˜2 − r1, and c1^3 = 2r2u˜2 + bϕ(uˆ1) − r2, 2˜3.

Du˜1 + eu˜2 − c = c3. Therefore, for any i between 1 and 3, ci ∈ C(D¯T). Applying the differential mean value theorem, we can readily deduce that, for every uˆ1 ≤ v1 ≤ u1 ≤ u˜1, f1(u1, u2, u3) − f1(v1, u2, u3) = r1(u1 − v1) − r1 (u1 + v1)(u1 − v1)

r1(u1 − v1) − r1 (u1 + v1)(u1 − v1) = 1^3 −(ϕ(u˜1) − ϕ(vˆ1))u2

1~3 −ϕ′(ξ)(u1 − v1)u2 ≥ (r1 − 2r1u˜1 − Lu˜2)(u1 − v1)

−c1(u1 − v1) = 1~3

where (v1, u1) is the range of ξ.

For uˆ2 ≤ v2 ≤ u2 ≤ u˜2, we can similarly derive that f2(u1, u2, u3) − f2(u1, v2, u3) = r2(u2 − v2) − r2 (u2 + v2)(u2 − v2)

2^3 + bϕ(u1)(u2 − v2) ≥ (r2 − 2r2u˜2 + bϕ(uˆ1))(u2 − v2), and for uˆ3 ≤ v3 ≤ u3 ≤ u˜3, f3(u1, u2, u3) − f3(u1, u2, v3) ≥ −c3(u3 − v3).

It is therefore demonstrated that for any uˆi ≤ vi ≤ ui ≤ u˜i, fi(ui, [u]ai, [u]bi) − fi(vi, [u]ai, [u]bi) ≥ −ci(ui − vi), where i = 1, 2, 3.(1.10) Consider the following: c1 = r1 − 2r1uˆ1, c2 = r2 − 2r2uˆ2 + bϕ(u˜1), c3 = c − du˜1 − euˆ2. As a result, for each i = 1, 2, 3, ci ∈ C(D¯T) k1+pu˜3~k2+qu˜3. Using the differential mean value theorem, we can quickly determine that, for each uˆ1 ≤ v1 ≤ u1 < u˜1, f1(u1, u2, u3) − f1(v1, u2, u3) = r1(u1 − v1) − r1 (u1 + v1)(u1 − v1)

One-third −(ϕ(uˆ1) − ϕ(vˆ1))r1 (u1 − v1) − r1 (u1 + v1)(u1 − v1) equals u2.

1~3 −ϕ′(ξ)(u1 − v1)u2 ≤ (r1 − 2r1uˆ1)(u1 − v1)

Where ξ ∈ (v1, u1), 1 3 = c1(u1 − v1).

For uˆ2 ≤ v2 ≤ u2 ≤ u˜2, we can similarly derive that f2(u1, u2, u3) − f2(u1, v2, u3) = r2(u2 − v2) − r2 (u2 + v2)(u2 − v2)

(r2 − 2r2uˆ2 + bϕ(u˜1))(u2 − v2) = 2^3 + bϕ(u1)(u2 − v2)

2^3 = c2(u2 − v2), and f3(u1, u2, u3) − f3(u1, u2, v3) < c3(u3 − v3) for uˆ3 ≤ v3 ≤ u3 ≤ u˜3.

Consequently, we establish that for each ci ∈ C(D¯T), such that uˆi ≤ vi ≤ ui ≤ u˜i,

(1.11)r1pu˜2 = fi(ui, [u]ai, [u]bi) − fi(vi, [u]ai, [u]bi) ≤ ci(ui − vi), i = 1, 2, 3.

For each i = 1, 2, 3, and K1,2 = ϕ(u˜1), K1,3 = r2qu˜2 ~1 2, K2,1 = bLu˜2, K2,3 = 1~3, let Ki,i = |ci| + |ci|.

2 2, Ki = Ki,1 + Ki,2 + Ki,3, i = 1, 2, 3, and K3,1 = du˜3 on D¯T. K3,2 = eu˜3. Thus, for any i, j = 1, 2, 3, 2^3 Ki,j ∈ C(D¯T) and Ki ∈ C(D¯T), indicating that Ki,j, and Ki are bounded functions in D¯T.

It derives from

for all i = 1 through 3. For u ∈ ▨uˆ, u˜▩,i = 1, 2, 3, this inequality demonstrates that fi meets the Lipschitz condition. Additionally, fi is a holder continuous function with i = 1, 2, and 3 on (t, x) ∈ D¯T.

The differential equations in system (1.7) can be expressed as (ui)t − Liui + ciui = Fi(ui, [u]ai, [u]bi), in DT, i = 1, 2, 3. Let Fi(ui,[u]ai,[u]bi) = fi(ui,[u]ai,[u]bi) + ciui, i = 1, 2, 3.

Lemma 8.1 in [39] gives us the lemma that follows.

Lemma 1.2. We indicate that Fi(u)(t, x) = Fi(u(t, x)) on D¯T for each u ∈ ▨uˆ, u˜▩, where i = 1, 2, 3. For all i = 1, 2, 3, the function Fi(u) is Holder continuous in DT if u ∈ Cα(DT) and α ∈ (0, 1). Furthermore, assuming that u ≥ v and that u, v ∈ ▨uˆ, u˜⟩, Fi(ui, [u]ai, [v]bi) − Fi(vi, [v]ai, [u]bi) ≥ 0, for i = 1, 2, 3.

Existence and Uniqueness of the Solution on ⟨ û , ũ ⟩

The upper solution u˝ = (u˝1, uˎ2, uˎ3) and lower solution uˆ = (uˆ1, uˆ2, uˆ3) of the system (1.7) are always assumed to exist in this section. For every i = 1, 2, 3, let Aiui = (ui)t − Liui + ciui. The two starting iterations are u¯(0) = uˎ and u¯(0) = uˆ. From the iteration process, we create the maximal and minimal sequences u¯(k) = (u¯(k), u¯(k), u¯(k)), u(k) = (u(k), u(k), u(k)).

1 2 3 1 2 3

In DT, where i = 1, 2, 3, i^i~i~i, Aiu¯(k) = Fi(u¯(k−1), [u¯(k−1)]a, [u(k−1)]b

Aiu(k) = Fi(u(k−1), [u(k−1)]a, [u¯(k−1)]b) in DT, i = 1, 2, 3, i~i~i~i Bu¯(k)(t, x) = Bu(k)(t, x) = 0 on ST, i = 1, 2, i~i u¯(k)(0, x) = u(k)(0, x) in Ω, i = 1, 2, 3.

I i

We first express the following positive lemmas, which were stated in [39], before demonstrating the monotone property of the maximal and minimal sequences.

Lemma 2.1: Let u ∈ C(D¯T) ∩ C1,2(DT) be such that ∂u − αΔu + βu ≥ 0, for all 0 < t ≤ T, x ∈ Ω,∂t ∂ u(t, x) ≥ 0, for all 0 < t ≤ T, x ∈ ∂Ω,∂v u(0, x) ≥ 0, for x ∈ Ω, where α > 0 and β = β(t, x) is a bounded function in DT = (0, T] × Ω. In DT, then u(t, x) >= 0. Furthermore, unless it is also zero in DT, u(t, x) > 0 in DT.

Lemma 2.2: Assume that u ∈ C(D¯T) ∩ C1,2(DT) and that for any t ≤ T, x ∈ Ω, ∂t u(0, x) ≥ 0, for x ∈ Ω, where β = β(t, x) is a bounded function in DT = (0, T] × Ω. In DT, then u(t, x) >= 0. Furthermore, unless it is also zero in DT, u(t, x) > 0 in DT.

We will now demonstrate the maximal and minimal sequences' monotone quality, which allows us to derive the following theorem.

Theorem 2.3. Let us assume the following: u^ = (uˎ1, uˎ2, uˎ3) and uˆ = (uˆ1, uˆ2, uˆ3) are Holder continuous in x, uniformly in DT; u1,0, u2,0 are Holder continuous on the domain Ω¯ satisfying the boundary condition at t = 0; u3,0 is Holder continuous on the domain Ω¯. Then, for any k, the maximal and minimal sequences {u¯(k)}, {u(k)} are well-defined on DT and have the monotone property uˆ ≤ u(k) ≤ u(k+1) ≤ u¯(k+1) ≤ u¯(k) ≤ u˜, (2.1) in DT. Additionally, u¯(k) and u(k) are connected upper and lower solutions of the system for each integer k (1.7).

Evidence. Since the proof for Theorem 2.3 is identical to that found in [31], it is not included here.

The pointwise (and componentwise) limits lim u¯(k)(t, x) = u¯(k)(t, x), lim u(k)(t, x) = u(k)(t, x) are due to the monotone property.(2.2)

In DT, k→∞k→∞ exist and meet the connection uˆ ≤ u ≤ u· ≤ u˜. We need to demonstrate that u ≤ u¯ in DT in order to demonstrate that the system (1.7) has a unique solution in ▨uˆ, u˜▩.

Remark 2.4. Assume that the system's connected upper and lower solutions are u˧ and uˆ (1.7). The system (1.7) has a unique solution, u∗, and u∗ ∈ ▨uˆ, u˜▩. Furthermore, the iteration process with starting iterations u¯(0) = u˜ and u(0) = uˆ yields sequences u¯(k), u(k) that both converge monotonically to u∗.

Evidence. Since the proof for Theorem 2.4 is identical to that found in [31], it is not included here.

- **Existence and Uniqueness of Solution of System (1.5)**

- First, we shall demonstrate that the system (1.5) on D¯T = [0, T] × Ω¯ has both upper and lower solutions. It is demonstrated that the system(1.5) solution exists on D¯T. Next, we'll demonstrate the system(1.5) solution's uniqueness on D¯T. As a result, the system (1.5) has a unique solution on D¯T, where T is any positive integer.

- Conclusion 3.1. Assume that constants α, β, M, and N meet the following conditions: α ≥ u3,0 ∞, β ≥ c, M ≥ max{⌷u1,0 ∞, k1 + pαeβT },(3.1)

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- The upper and lower coupled solutions of the system (1.7) on [0, T] × Ω¯ are then two functions (u˜1, u˧2, u˜3) = (M, N, αeβt), (uˆ1, uˆ2, uˆ3) = (0, 0, 0). Moreover, Holder continuous in x, uniformly in D¯T, are u˜ = (u˜1, u˒2, u˜3) and uˆ = (uˆ1, uˆ2, uˆ3).

- Evidence. We have uˆ1(t, x) = 0 ≤ M = u˜1(t, x), uˆ2(t, x) = 0 ≤ N = u˧2(t, x), uˆ3(t, x) = 0 ≤

{∂v 2 {∂v 2 because u˜1 = M, uˆ1 = 0, u˧2 = N, uˆ2 = 0, and u˧3 = αeβt, uˆ3 = 0.

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- uˆ1(0, x) = 0 ≤ u1,0(x) ≤ max{u1,0(x), x ∈ Ω¯ } ≤ M = u˜1(0, x) is the result of M ≥ u1,0 ∞ = max{u1,0(x), x ∈ Ω¯ }, uˆ1 = 0.

- Similarly, we have uˆ2(0, x) = 0 ≤ u2,0(x) < max{u2,0(x), x ∈ Ω¯ } ≤ M = u˜2(0, x) from N ≥ u2,0 ∞ = max{u2,0(x), x ∈ Ω¯ }, uˆ2 = 0.

- We can determine that uˆ3(0, x) = 0 ≤ u3,0(x) ≤ max{u3,0(x), x ∈ Ω¯ } ≤ α = u˜3(0, x) from α ≥ u3,0 ∞ = max{u3,0(x), x ∈ Ω¯ }, uˆ3 = 0.

- Thus, the upper and lower coupled solutions of the system (1.7) on [0, T] × Ω¯ are represented by a pair of functions (u˜1, u˧2, u˜3) = (M, N, αeβt), (uˆ1, uˆ2, uˆ3) = (0, 0, 0). The proof is finished.

- The following result is easily obtained from Theorem 3.1.

- Conclusion 3.2. Assume α, β, M, and N are constants that fulfil α ≥ w0 ∞, β ≥ c, M ≥ max{⌷u0 ∞, k1 + pαeβT },

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- N = max{⁞v}, r2 + bϕ(M)(k}+ qαe\T)}, (3.2)

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- Afterwards, the upper and lower coupled solutions of the system (1.5) on [0, T] × Ω¯ are represented by a pair of functions (u˜, v˜, w˜) = (M, N, αeβt), (uˆ, vˆ, wˆ) = (0, 0, 0). Moreover, Holder continuous in x, uniformly in D¯T, are (u˒, v˜, w˜) and (uˆ, vˆ, wˆ).

- We shall now demonstrate that the unique solution 1~2~3 of the system (1.7) on [0, T] × Ω¯ is the limit of the maximum and minimum sequences u¯(k) = (u¯(k), u¯(k), u¯(k)), u(k) = 1~2~3 (u(k), u(k), u(k)) with initial iterations u¯(0) = (M, N, αeβt) and u(0) = (0, 0, 0).

- Theorem 3. Assuming (3.1) is true, there exists a unique solution (u1, u2, u3) for the system (1.7) on [0, T] × Ω¯, and (0, 0, 0) ≤ (u1, u2, u3) ≤ (M, N, αeβt).

- Evidence. Assuming that the solutions of the system (1.7) on [0, T] × Ω¯ are (u1, u2, u3) and (v1, v2, v3), there exists a positive number M0 such that (0, 0, 0) < (u1, u2, u3), and (v1, v2, v3) ≤ (M0, M0, M0).

- A comparable demonstration of (1.12) in Section 1 allows us to quickly determine that constants K˜i, where i = 1, 2, and 3, exist such that

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- Given that {|fi(u1, u2, u3) − fi(v1, v2, v3)| ≤ K˜i(|u1 − v1| + |u2 − v2| + |u3 − v3|).

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- (u1)t − d1∆u1 = f1(u1, u2, u3), (u2)t − d2∆u2 = f2(u1, u2, u3), (u3)t = f3(u1, u2, u3), (v1)t − d1∆v1 = f1(v1, v2, v3), (v2)t − d2∆v2 = f2(v1, v2, v3), (v3)t = f3(v1, v2, v3),

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- as well as the starting and boundary conditions

- ∂ v (t, x) = 0, x ∈ ∂Ω, t > 0, i = 1, 2, ∂v i˜∂v i ui(0, x) = vi(0, x) = ui,0(x), x ∈ Ω, i = 1, 2, 3.

- For every t in [0, T] and every x in Ω¯, let ui(t)(x) = ui(t, x) and vi(t)(x) = vi(t, x). We can readily obtain that u1(t) − v1(t) + u2(t) − v2(t) + u3(t) − v3(t) = u1(s) − v1(s) + u2(s) − v2(s) + u3(t) − v3(t) ∞ t ≤ (K1 + K2 + K3)~(u1(s) − v1(s) + u2(s) − v2(s) + u3(s) − v3(s))ds.

- Gronwall's inequality thus gives us ui(t, x) = vi(t, x), i = 1, 2, 3, for any t in the interval [0, T] and x in the interval Ω¯. The system (1.7) has a unique solution (u1, u2, u3) on [0, T] × Ω¯, and (0, 0, 0) ≤ (u1, u2, u3) ≤ (M, N, αeβt), according to Theorems 2.4 and 3.1.

- The following result is easily obtained from Theorem 3.3.

- Theme 3.4. If equation (3.1) is true, then there is only one solution (u, v, w) for the system (1.5) on [0, T] × Ω¯, and (0, 0, 0) ≤ (u, v, w) ≤ (M, N, αeβt).

Conclusion

This research examines a modified Lesie-Gower food web model with three species, featuring omnivory—defined as feeding on more than one trophic level—and general nonlinear functional response.

degree. The biotic resource's population size plus a constant determines the model's carrying capacity. Jau also examined a three-species Lesie-Gower food web model (1.3) in [11] using the same biotic resource; however, the model's carrying capacity is only proportionate to the biotic resource's population size without the addition of a constant. At low densities, it exhibits slightly unique behaviour, which makes it impossible to linearize the model at the border equilibria. In fact, the analysis of the system is greatly complicated by this singularity (1.3). As a result, the model takes omnivory into account (1.5). Using the techniques of upper and lower solutions along with semigroup theory, we can determine that if (3.1) is true, then there is only one solution (u, v, w) for the system (1.5) on [0, T] × Ω¯ and (0, 0, 0) ≤ (u1, u2, u3) ≤ (M, N, αe\t).

According to Jau's findings in [11], if (1.4) is true, then there exists a unique solution (u, v, w) for the system (1.3) on [0, T] × Ω¯, and (0, 0, εe−Kt) < (u, v, w) ≤ (M, N, αeβt).

Clearly, compared to condition (1.4), condition (3.1) is weaker and simpler. Additionally, our paper's scope of solutions is greater than that of [11]. From the explanation above, it is clear that omnivory has a significant impact on the system's uniqueness and existence (1.5). Indeed, several Artiodactyla species are omnivores; they include wild boar, which mostly eats grass, sweet potatoes, roots, tubers, and wild fruit. Because they can consume a variety of foods, if one type of food is scarce, like wild fruit, they can eat sweet potatoes instead, which will help them survive in the harsh environment.

Indeed, the system (1.5) can be changed to the system (1.3) if ki → 0, i = 1, 2, and ϕ(u) = au. As a result, when examining system (1.5), we can clearly observe more dynamic behaviours of system (1.3). One of the primary findings of Jau's study [Jau GC. The challenge of the nonlinear diffusive predator-prey model with the same biotic resource] is demonstrated to be complemented and enhanced by our result. Real World Applications, Nonlinear Anal., 2017; 34: 188-200].

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